ISSN: 1803-3814 (Printed), 2571-3701 (Online)
https://doi.org/10.13164/mendel.2021.1.041

Mixed-Integer Programming Model for Ranking Universities: Letting Universities Choose the Weights

Jakub Kudela✉

Institute of Automation and Computer Science, Brno University of Technology, Czech Republic
Jakub.Kudela@vutbr.cz

Abstract

Regardless of the shortcomings and criticisms of world university rankings, these metrics are still widely used by students and parents to select universities and by universities to attract talented students and researchers, as well as funding. This paper proposes a new mixed-integer programming model for ranking universities. The new approach alleviates one of the criticisms – the issue of the "arbitrariness" of the weights used for aggregation of the individual criteria (or indicators) utilized in the contemporary rankings. Instead, the proposed model uses intervals of different sizes for the weights and lets the universities themselves "choose" the weights to optimize their position in the rankings. A numerical evaluation of the proposed ranking, based on the indicator values and weights from the Times Higher Education World University Ranking, is presented.

Keywords: *ranking, university ranking, mixed integer programming, multiple-criteria decision-making.*

Received: 10 May 2021
Accepted: 15 June 2021
Published: 21 June 2021

1 Introduction

The ranking of universities has in recent years been used as an important tool for universities to publicise their prestige and international positioning [37]. Since the conception of the Academic Ranking of World Universities (ARWU) [23], both the number of university rankings and the number of universities included in them have steadily increased. The growth of these rankings as well as the increase in universities interested in them is explained by the interest of different groups [16].

Universities themselves are eager to occupy dominant positions in these rankings, since this is a way of getting the attention of a greater number of potential students, increasing the revenues from student enrolment [14], and the rankings themselves have an irreducible reputation-making role [26]. The practice of ranking universities has become widely defined by national and international organisations as an important instrument of political and economic policy [3]. This is clearly reflected when ranking positions determine policies related to the restructuring of the higher education system, as in the French case, where different universities were merged to create a university with a high ranking [6]. Despite the shortcomings and the criticism of the university rankings, they are still widely used by students and their parents to select institutions, and by educational institutions to attract talented students and researcher, as well as funding [24]. According to [4], funding explains up to 51% of the variability of the positions attained by the universities in some rankings.

Apart from the AWRU ranking (or, commonly known as Shanghai ranking), the other prominent rankings are the Times Higher Education (THE) World University Ranking , and the Quacquarelli Symonds (QS) ranking. All three rankings are based on weighing a set of indicators (or dimensions) that should adequately describe the performance of a university in different areas. These indicators typically consist of the quality of education, quality of faculty, number of citations, industry income, international outlook, etc. Recently, new rankings have focused on prominent dimensions not explicitly addressed in the above-mentioned rankings, such as innovation (Scimago Institutions Rankings— SIR), web visibility [22], and impact (Webometrics Ranking), or sustainability (GreenMetric World University Ranking). The authors of [10] showed how the intelligent integration of existing data about universities may lead to an open-linked data platform which permits the construction of new indicators, which combine heterogeneous sources of data to generate indicators that address a variety of user requirements without the need to design indicators on a custom basis.

Several researchers have outlined challenging concerns that should be considered before assessing the performance of universities. For instance, some rankings that include survey-based information may bias the results towards those universities that are well-known compared with lesser-known ones [35]. A factor analysis of the ARWU, QS and THE rankings was performed in [34] and revealed that there were two factors in each of the ranking systems, which did not support the assumption that the indicators were mutually supporting and additive as conceptualised by the ranking providers. The leading global university rankings were also explored in [28] to determine the similarities and differences in terms of their ranking criteria, main indicators, modeling choices, and the effects of these on the rankings. In [30], the authors used a sample of Scandinavian universities to show that the differences between the THE and ARWU rankings may be attributed to both small variations on what they believe are not important indicators, as well as substantial variations on what they believe are important indicators. They also provide a methodology that can be used in understanding universities' different ranks in global university rankings. In [7], the authors studied governmentalities of globalizing higher education through a discussion of the competing logics and landscapes of reputation and ranking in two leading universities in South Korea. Their analysis draws attention to the ways in which university rankings have generated a new multi-scalar geography of institutional reputation, the mismatch between quality, reputation and ranking, and the new kinds of institutional behaviors that are emerging to respond to the proliferation of ranking systems. A comparative analysis o five world university rankings was carried out in [27], where it is argued that current rankings are still one-dimensional in the sense that they provide finalized, seemingly unrelated indicator values. The authors of [17] argue that compared to other ranking tools like THE and QS, the ARWU ranking is in many ways "better" – it is simple and transparent, relying on public knowledge and third-party data, not on data provided by the universities themselves or solely on performance data provided by operators like the Institute for Scientific Information. The utilization of rankings and indicators within the universities can also be used as an instrument of new managerialism [25].

Even considering the above shortcomings, there is still a wide consensus that the main weakness of the aforementioned university rankings is linked to the determination of the weights used to measure both dimensions and indicators in the computation of university performance [14]. The combination of multiple indicators of university performance in a single aggregate measure is usually carried out in a rather arbitrary way, which prevents a clear interpretation of the aggregated measure [31, 35]. Also, Using unstandardized indicators for the total scores used in ranking can lead to undesirable results [33]. In the last decade, there have been several approaches proposed to alleviate the aforementioned problems. In [8] the authors proposed a robust ranking of universities, where the aggregation of the university performances is be done by the Choquet integral preference model that is able to take into account the possible negative and positive interactions between the different criteria. A ranking methodology based on ranking hesitant fuzzy sets was developed in [2]. The authors of [29] proposed to measure the importance of a given variable within existing composite indicators via Karl Pearson's "correlation ratio". Hybrid multi-criteria decision making was utilized for ranking 12 private universities in Taiwan in [37]. The composite I-distance indicator methodology as an alternative to weighting was proposed in [12]. In [24], the authors used a contrast pattern mining algorithm to extract a set of patterns describing the top 100 universities in the QS World University Rankings and showed, how are these top universities separable from the rest. A robustness analysis, based on a multi-modelling approach, was performed in [32] to test the validity of the inference about the rankings produced in the ARWU, Shanghai, and THE rankings. The authors conclude that while university and country level statistical inferences are unsound, the inference on macro regions is more robust, and propose an alternative ranking. In [11], the authors proposed a conditional multidimensional approach based on a robust directional distance technique for ranking European universities in teaching and research activities. A goal programming model for the ranking of universities was developed in [14]. Data envelopment analysis model for jointly evaluating the relative teaching and research efficiencies of universities was presented in [18].

In this paper, we propose a new mixed-integer programming model for ranking universities. The model is unique in that it allows each university to "choose" the weights for the aggregation of the individual criteria to get the best possible ranking. It alleviates the issue of of the "arbibtrariness" of the particular values of weights used in different rankings and allows instead for intervals of different sizes for the weights from which the universities can "choose" to optimize their position in the ranking.

The rest of the paper is organised in the following manner. Section 2 introduces the mixed-integer programming model framework for computing the ranking of universities. Section 3 briefly describes the dataset used for the numerical evaluation of the model. Section 4 presents and discusses the empirical results. The main conclusions and implications of the paper are presented in Section 5.

2 Mathematical Model

Mixed-integer programming (MIP) is one of the most ubiquitous modelling methods used in optimization [9, 20, 36], with applications ranging from optimal social distancing [19] to optimal plan for the construction of waste processing plants [21].

Our MIP model for ranking universities is based on the following idea: If we let the universities themselves decide on the values of the weights for the individual indicators, how would they choose? Let us consider a situation, where there are N universities to be ranked according to D criteria (or indicators). The values of these criteria are already known and denoted by a vector $p_i \in \mathcal{R}^D$ for each university i. In order to obtain the ranking, we need to determine the weights $w \in \mathcal{R}^D$ of the criteria to get a overall score. If we let a particular university k choose these weights, it will naturally set them in such a way that its own ranking is as good as possible. This can be achieved by solving the following MIP problem, where w^k denotes that the weights are chosen by university k:

$$\begin{aligned}
\text{minimize} \quad & N - \sum_{i=1}^{N} y_i && (1)\\
\text{subject to} \quad & p_k' w^k \leq p_i' w^k + M y_i, \quad \forall i \neq k, && (2)\\
& p_k' w^k \geq p_i' w^k - M(1 - y_i), \quad \forall i \neq k, && (3)\\
& \sum_{j=1}^{D} w_j^k = 1, && (4)\\
& y_k = 0, && (5)\\
& l_j \leq w_j^k \leq u_j, \quad \forall j, && (6)\\
& y_i \in \{0, 1\}, \quad \forall i. && (7)
\end{aligned}$$

The objective function (1) describes the position of the university k withing the ranking (which should be minimized). The binary variable y_i decodes, if the university i is ranked worse than university k ($y_i = 1$) or not ($y_i = 0$). This relationship is enforced by the so-called "Big-M" constraints (2)-(3). The value of the parameter M should be large enough so the conditions hold, but not too large, as it might bring numerical difficulties. Constraint (4) is a normalizing condition on the weights. Constraint (5) forces the ranking to start from position 1 (otherwise, the best ranking university would have position 0). And, finally, constraints (6) and (7) enforce that the weights for the indicators are within pre-specified bounds, and that the variables y_i are binary.

After solving the MIP problem (1)-(7) we get the best possible position of the university k (the value of the objective function), and, more importantly, the optimal weights w^k, which also determine the ranking of all other universities. If we solve MIP problem (1)-(7) for all N universities, we effectively get N observations of possible rankings from which we can easily extract meaningful statistical results. This approach alleviates the issue of the "arbitrariness" of the weights used in the different rankings – instead of a single value, the individual indicators can have a range of values, and the resulting ranking is left "on the universities themselves".

3 Data

This section presents the database used to illustrate the implementation of the aforementioned university ranking. Although there is a large variety of rankings currently in use, we chose to apply our model to the data provided by the THE ranking [1]. This ranking is among those with the largest historical data, the number of universities listed in this ranking is very large, and it has the important data readily available at their website.

Table 1 contains a list of the indicators and the weights used in the THE ranking in years 2018-2021. The values of the indicators are standartized: the procedure is based on the distribution of data within a particular indicator, where a cumulative probability function is calculated, using a version of Z-scoring [1]. This means that all values of the indicators fit in a range between 0 (worst performing universities in the indicator) and 100 (best performing universities). The values of the five indicators indicators and their weighted average (Overall) for the top 10 universities in 2021 can be found in Table 2. The values of the indicators for all considered universities in years 2018-2021 can be found in the supplementary file "DATA.xls".

Table 1: List of indicators used in the THE ranking. [1]

Indicator	Definition	Weight
International outlook	international-to-domestic-student ratio international-to-domestic-staff ratio international collaboration	7.5%
Industry income	knowledge transfer	2.5%
Teaching	reputation survey staff-to-student ratio doctorate-to-bachelor's ratio doctorates awarded-to-academic staff ratio institutional income	30%
Research	reputation survey research income research productivity	30%
Citations	research influence	30%

4 Empirical Results

We use the indicator values from the THE ranking as a ground for the empirical evaluation of the proposed ranking. The range for the weights is also based on the THE ranking (Table 1) and denote them by w^T. We use a parameter α to denote a possible deviation from the base values, which results in the equation (6) having the following form:

$$(1-\alpha) w_j^T \leq w_j \leq (1+\alpha) w_j^T, \quad \forall j.$$

We compute the results for three different values of $\alpha = [0.3, 0.2, 0.05]$ and four years: in 2021 there were $N = 1527$ universities, in 2020 there were $N = 1397$ universities, in 2019 there were $N = 1258$ universities, and in 2018 there were $N = 1103$ universities considered in the THE ranking. Since the values of the indicators are within 0 and 100, their weighted average will also lie within these abounds. This means that we can set to the value of the "Big M" parameter to $M = 100$.

The optimization model was programmed in the high-performance dynamic language JULIA [5] with the JuMP package for mathematical optimization [13]. The solution was computed by the GUROBI 8.0 solver [15]. The computations were carried out on an ordinary computer (3.2 GHz i5-4460 CPU, 16 GB RAM) and took around five minutes finish for one instance

Table 2: Indicator and Overall values for the top 10 universities in 2021. [1]

Rank	Name	Overall	Teaching	Research	Citations	Industry income	International outlook
1	University of Oxford	95.6	91.3	99.6	98.0	68.7	96.4
2	Stanford University	94.9	92.2	96.7	99.9	90.1	79.5
3	Harvard University	94.8	94.4	98.8	99.4	46.8	77.7
4	California Institute of Technology	94.5	92.5	96.9	97.0	92.7	83.6
5	Massachusetts Institute of Technology	94.4	90.7	94.4	99.7	90.4	90.0
6	University of Cambridge	94.0	90.3	99.2	95.6	52.1	95.7
7	University of California, Berkeley	92.2	85.8	97.2	99.1	84.3	72.3
8	Yale University	91.6	91.9	93.8	97.9	56.1	68.4
9	Princeton University	91.5	88.8	92.5	98.9	58.0	80.2
10	The University of Chicago	90.3	88.9	90.5	98.6	54.9	74.0

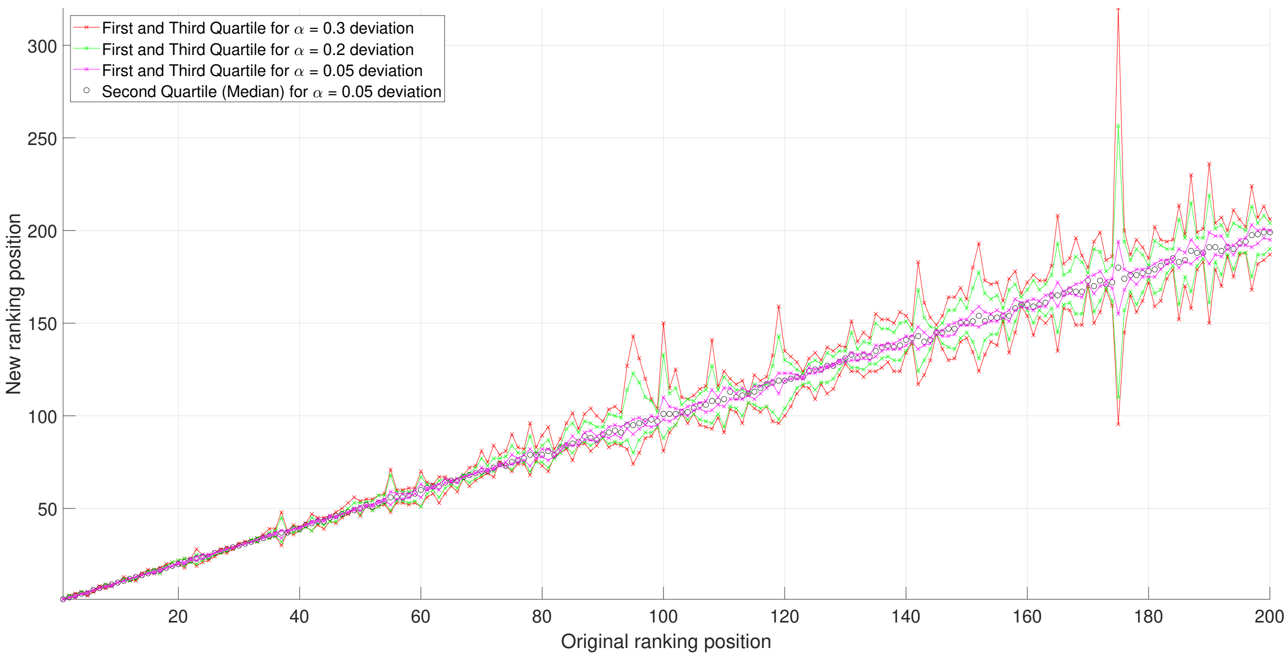

Figure 1: Results of the first 200 universities for different values of α, year 2021.

Table 3: Indicator and Overall values for selected universities in 2021. [1]

Rank	Name	Overall	Teaching	Research	Citations	Industry Income	International Outlook
91	University of Bristol	63.0	40.3	48.6	95.6	39.8	88.2
92	University of Basel	62.9	44.0	41.4	91.7	99.7	97.2
92	University of Glasgow	62.9	40.9	48.1	94.3	39.9	92.8
94	Purdue University West Lafayette	62.5	57.1	65.5	62.0	69.9	71.5
94	Zhejiang University	62.5	65.9	65.6	52.3	100.0	65.1
96	Korea Advanced Institute of Science and Technology	62.4	64.4	68.1	57.9	100.0	36.6
97	National Taiwan University	62.3	57.1	66.7	66.9	69.5	44.8
170	University of Leicester	56.0	30.9	33.6	96.2	37.6	90.7
170	University of Notre Dame	56.0	52.1	45.5	71.5	38.1	57.3
170	Sant'Anna School of Advanced Studies – Pisa	56.0	45.8	39.6	79.9	85.9	57.2
174	University of Exeter	55.9	32.4	38.3	89.9	35.8	91.5
174	Lomonosov Moscow State University	55.9	80.0	67.6	12.9	97.7	70.7
176	Northeastern University	55.8	36.7	29.1	98.2	36.5	76.0
176	Ulsan National Institute of Science and Technology	55.8	34.9	40.7	90.8	85.3	49.1
178	University of Aberdeen	55.7	29.5	34.2	94.4	45.1	95.6
178	Newcastle University	55.7	31.4	38.2	90.6	40.1	88.0
178	Paris-Saclay University	55.7	37.3	48.7	80.4	34.5	65.1

(solving (1)-(7) for one range on the weights, one year, N times for all universities).

As the relevant statistical information about the resulting ranking of a given university, we chose the first quartile, the second quartile (the median), and the third quartile of its position (out of the N possible rankings that were "chosen" by the N universities). The detailed results of the computations can be found in the supplementary file "RESULTS.xls". In Fig. 1 are shown the statistical information about the first 200 universities for the year 2021 and different values of α. We can see that, roughly speaking, the high ranking universities retain their high ranking and have a relatively small difference between the first and the third quartile of their position, even for the largest considered deviation. As the position of the university goes down, this interval between the quartiles widens. An interesting jump in the size of the interval can be seen around universities with original THE ranking position between 91 and 97. The values of the indicators for these universities are reported in Table 3. The thing the universities with large intervals have in common is a relatively lower value of the "Citations" indicator when compared with other similarly ranked universities (which is compensated by their relatively higher values in "Teaching" and "Research"). Similar, but a bit more dramatic effect can be seen between the positions 170 and 180, where the " Lomonosov Moscow State University" stands out with a large difference between the first and third quartile, which is caused by its poor value of the "Citations" indicator.

Although the size of the interval is higher for lower ranking universities, it does not grow to unreasonable values. The sizes of the intervals for different values of α and different years can be seen in Fig. 2. Naturally, the higher the allowed deviation α, the larger the size of the intervals. The THE ranking reports a "precise" rank for the first 200 universities and groups the rest into ranges of 201-250, 251-300, 301-350, 351-400, 401-500, 501-600, 601-800, 801-1000, and 1000+ (i.e., 1001-1527 for the year 2021). The ranking proposed obtained by the proposed method results in a smaller ranges, as can be seen in Table 4. The effect of the proposed ranking on the top 100 universities in years 2018-2021 can be seen in Table 5.

5 Conclusion

Despite their problems and criticisms, rankings of the world universities are here to stay, and will go on to be used by students, parents, researchers, and funding agencies, for categorizing and selecting universities. In this paper, we proposed a mixed-integer programming model for ranking universities that aims to alleviate one of the criticisms faced by contemporary university rankings – the "arbitrariness" of the weights used for aggregate scores. We have shown, that by using intervals for the weights instead and letting the universities themselves choose the particular values of the weights from these intervals to optimize their position in the rankings, we can get a reasonable ranking. We have demonstrated the properties of the model on a numerical example that was based on the THE ranking in years the 2018-2021. The top-performing universities still occupy the best positions regardless of the approach followed by the mixed-integer programming model, confirming their leadership. For the other universities, the mixed-integer model provides a meaningful range on their ranking based on the difference between the third and first quartile of their position in all possible rankings (chosen by all the universities). These ranges are comparatively smaller than the ones reported in the THE ranking, even for large values of the allowed deviation α. Although the "arbitrariness" of the weights is alleviated, it is not completely removed, as the intervals for the possible values of the weights still have to be agreed upon.

Acknowledgement: This work was supported by IGA Brno University of Technology project No. FSI-S-20-6538.

References

[1] Times higher education university ranking, 2021. https://www.timeshighereducation.com/world-university-rankings/ [Accessed 26th Apr 2021].

[2] Alcantud, J., de Andrés Calle, R., and Torrecillas, M. Hesitant fuzzy worth: An innovative ranking methodology for hesitant fuzzy subsets. *Applied Soft Computing 38* (2016), 232–243.

[3] Amsler, S., and Bolsmann, C. University ranking as social exclusion. *British Journal of Sociology of Education 33* (2012), 283–301.

[4] Benito, M., Gil, P., and Romera, R. Funding, is it key for standing out in the university rankings? *Scientometrics 121* (2019), 771–792.

[5] Bezanson, J., Edelman, A., Karpinski, S., and Shah, V. Julia: a fresh approach to numerical computing. *SIAM Review 59* (2017), 65–98.

[6] Billaut, J., Bouyssou, D., and Vincke, P. Should you believe in the shanghai ranking? *Scientometrics 84* (2009), 237–263.

[7] Collins, F., and Park, G. Ranking and the multiplication of reputation: reflections from the frontier of globalizing higher education. *Higher Education 72* (2016), 115–129.

[8] Corrente, S., Greco, S., and Słowiński, R. Robust ranking of universities evaluated by hierarchical and interacting criteria. In *Huber S., Geiger M., de Almeida A. (eds) Multiple Criteria Decision Making and Aiding. International Series in Operations Research & Management Science* (2019), pp. 145–192.

[9] da Silva, R. F., Hvattum, L. M., and Glover, F. Combining solutions of the optimum satisfiability problem using evolutionary tunneling. *MENDEL Journal 26* (2020), 23–29.

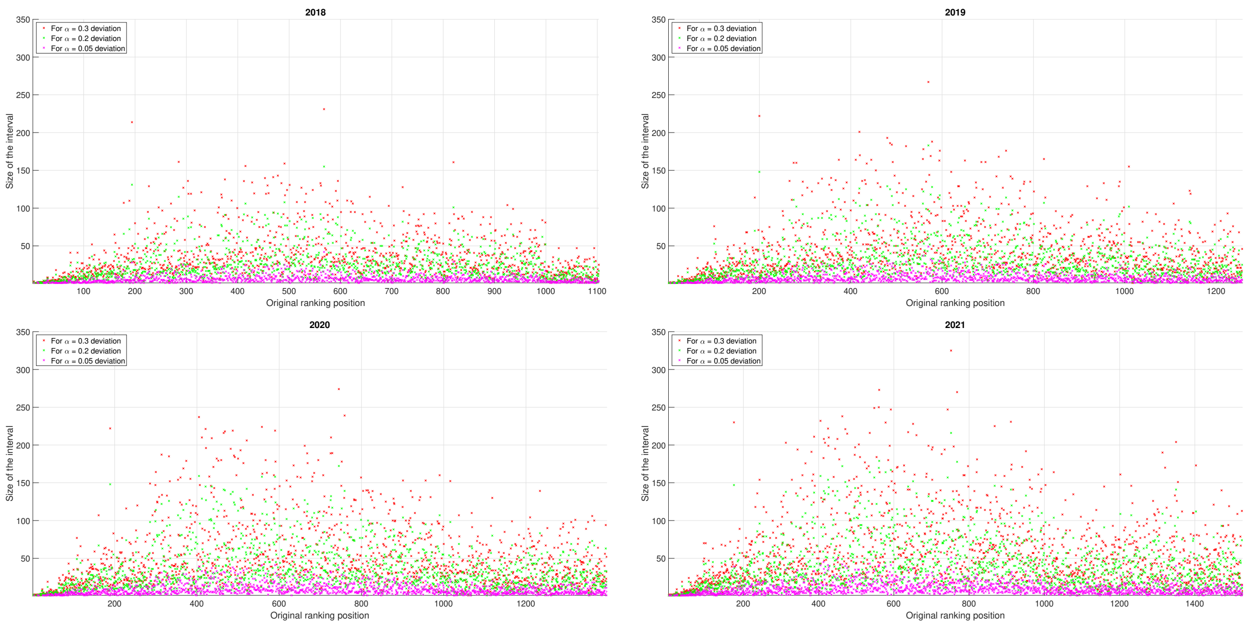

Figure 2: Size of the interval between the first and the third quartile for different values of α and different years.

Table 4: Interval sizes (difference between the first and third quartile) for different values of α and different years.

	median interval size			maximum interval size		
	$\alpha = 0.3$	$\alpha = 0.2$	$\alpha = 0.05$	$\alpha = 0.3$	$\alpha = 0.2$	$\alpha = 0.05$
2018	28	19	5	231	155	42
2019	34	22	6	267	183	49
2020	39	25	6	274	172	41
2021	44	29	7	325	216	51

[10] Daraio, C., and Bonaccorsi, A. Beyond university rankings? generating new indicators on universities by linking data in open platforms. *Journal of the Association for Information Science and Technology 68* (2017), 508–529.

[11] Daraio, C., Bonaccorsi, A., and Simar, L. Rankings and university performance: A conditional multidimensional approach. *European Journal of Operational Research 244* (2015), 918–930.

[12] Dobrota, M., Bulajic, M., Bornmann, L., and Jeremic, V. A new approach to the qs university ranking using the composite i-distance indicator: Uncertainty and sensitivity analyses. *Journal of the Association for Information Science and Technology 67* (2016), 200–211.

[13] Dunning, I., Huchette, J., and Lubin, M. Jump: a modeling language for mathematical optimization. *SIAM Review 59* (2017), 295–302.

[14] García, F., Guijarro, F., and Oliver, J. A multicriteria goal programming model for ranking universities. *Mathematics 9* (2021), 459.

[15] GUROBI. Gurobi optimizer 8.0, 2021. http://www.gurobi.com/ [Accessed 26th Apr 2021].

[16] Johnes, J. University rankings: What do they really show? *Scientometrics 115* (2018), 585–606.

[17] Kauppi, N. The global ranking game: narrowing academic excellence through numerical objectification. *Studies in Higher Education 43* (2018), 1750–1762.

[18] Kuah, C., and Wong, K. Efficiency assessment of universities through data envelopment analysis. *Procedia Computer Science 3* (2011), 499–506.

[19] Kudela, J. Social distancing as p-dispersion problem. *IEEE Access 8* (2020), 149402–149411.

[20] Kudela, J., and Popela, P. Two-stage stochastic facility location problem: Ga with benders decomposition. *MENDEL 21* (2015), 53–58.

[21] Kudela, J., Smejkalova, V., Somplak, R., and Nevrly, V. Legislation-induced planning of waste processing infrastructure: A case study of the czech republic. *Renewable and Sustainable Energy Reviews 132* (2020), 110058.

[22] Lages, J., Patt, A., and Shepelyansky, D. Wikipedia ranking of world universities. *The European Physical Journal B 89* (2016), 69.

[23] Liu, N., and Cheng, Y. The academic ranking of world universities. *Higher Education in Europe 30* (2005), 127–136.

[24] Loyola-González, O., Medina-Pérez, M., Valdez, R., and Choo, K.-K. A contrast pattern-based scientometric study of the qs world university ranking. *IEEE Access 8* (2020), 206088–206104.

Table 5: The values of the first, the second (in bold), and the third quartile for the top 100 universities in years 2018-2021, allowed deviation $\alpha = 0.3$.

	2021	2020	2019	2018		2021	2020	2019	2018
University of Oxford	1:**1**:1	1:**1**:1	1:**1**:1	1:**1**:1	UoM	51:**51**:54	56:**55**:58	55:**56**:58	53:**54**:57
Stanford University	2:**2**:3	2:**4**:4	2:**3**:3	3:**3**:3	University of Sydney	49:**51**:55	56:**60**:61	58:**59**:64	60:**61**:65
Harvard University	2:**3**:4	5:**7**:7	5:**6**:6	6:**6**:6	UoSC	52:**53**:57	62:**62**:65	62:**66**:68	60:**66**:68
CalTech	4:**4**:5	2:**2**:3	3:**5**:5	2:**3**:5	Boston University	52:**54**:58	59:**61**:64	68:**74**:83	64:**70**:77
MIT	3:**5**:5	5:**5**:5	4:**4**:5	4:**5**:5	Kyoto University	47:**54**:71	52:**65**:76	54:**65**:88	58:**74**:99
University of Cambridge	5:**6**:6	2:**3**:4	2:**2**:3	2:**2**:4	CUoHK	53:**56**:60	54:**57**:59	51:**53**:55	58:**58**:61
UoCB	7:**7**:8	13:**13**:14	14:**15**:15	17:**18**:19	THKUoSaT	54:**56**:60	44:**47**:49	40:**41**:46	41:**44**:46
Yale University	7:**8**:9	8:**8**:8	8:**8**:8	10:**12**:12	UoNCaCH	52:**56**:61	54:**54**:59	55:**57**:59	53:**56**:62
Princeton University	8:**9**:9	6:**6**:6	7:**7**:7	7:**7**:7	ANL	54:**59**:61	48:**50**:53	47:**49**:52	46:**48**:50
The University of Chicago	10:**10**:10	9:**9**:10	9:**10**:10	8:**9**:9	SNU	51:**60**:69	54:**64**:71	56:**63**:77	63:**74**:91
Imperial College London	11:**11**:13	9:**10**:11	9:**9**:10	8:**8**:9	Brown University	57:**61**:64	53:**53**:58	51:**53**:54	47:**50**:52
Johns Hopkins University	11:**12**:12	11:**12**:12	11:**12**:12	13:**13**:14	TUoQ	59:**62**:63	65:**66**:68	67:**69**:74	63:**65**:66
University of Pennsylvania	11:**13**:13	10:**11**:12	11:**13**:13	10:**10**:11	WU&R	54:**62**:67	55:**59**:63	57:**59**:66	59:**64**:68
ETH Zurich	14:**14**:15	13:**13**:14	11:**11**:13	10:**10**:12	UoCD	58:**64**:67	55:**55**:59	61:**59**:62	53:**54**:55
UoCLA	15:**15**:17	15:**17**:17	17:**17**:17	14:**15**:16	Monash University	62:**64**:65	75:**75**:82	82:**84**:89	80:**80**:82
University College London	15:**16**:17	15:**15**:17	14:**14**:15	15:**16**:17	University of Amsterdam	60:**66**:66	61:**62**:65	59:**62**:65	56:**59**:64
Columbia University	15:**17**:18	15:**16**:17	15:**16**:16	13:**14**:15	UNSW Sydney	66:**67**:68	69:**71**:74	90:**96**:99	81:**85**:90
University of Toronto	18:**18**:20	18:**18**:19	20:**21**:22	22:**22**:23	UoCSB	62:**68**:72	53:**57**:61	47:**52**:56	49:**53**:59
Cornell University	19:**19**:21	19:**19**:19	18:**19**:19	18:**19**:19	McMaster University	65:**69**:73	67:**72**:83	70:**77**:90	76:**78**:85
Duke University	20:**20**:23	20:**20**:23	18:**18**:19	16:**17**:17	Fudan University	66:**70**:80	89:**109**:125	87:**104**:126	94:**116**:146
Tsinghua University	18:**20**:23	20:**23**:27	18:**22**:28	25:**30**:33	Leiden University	69:**70**:75	66:**67**:70	66:**68**:71	67:**67**:72
UoMAA	22:**22**:23	21:**21**:22	20:**20**:21	21:**21**:23	EUR	67:**72**:85	64:**69**:79	63:**70**:80	66:**72**:78
Peking University	19:**23**:28	20:**24**:29	25:**31**:34	25:**27**:33	University of Montreal	75:**73**:79	82:**85**:93	88:**90**:95	104:**108**:113
Northwestern University	21:**24**:25	21:**22**:24	23:**25**:26	20:**20**:22	University of Zurich	73:**73**:82	86:**90**:100	87:**92**:99	124:**136**:145
NUoS	22:**25**:25	24:**25**:28	23:**23**:27	20:**22**:24	CUB	71:**75**:91	73:**80**:95	83:**90**:99	117:**126**:136
New York University	24:**26**:26	26:**29**:28	25:**27**:28	27:**27**:29	Utrecht University	74:**75**:84	74:**75**:83	69:**74**:81	68:**68**:72
LSoEaPS	26:**27**:28	25:**27**:27	24:**26**:27	26:**26**:29	University of Warwick	75:**77**:84	74:**77**:85	76:**79**:87	88:**90**:96
Carnegie Mellon University	26:**28**:29	24:**27**:29	22:**24**:26	22:**24**:25	DUoT	68:**78**:95	60:**67**:81	55:**58**:64	58:**63**:73
University of Washington	28:**29**:29	24:**26**:26	26:**28**:29	26:**25**:28	University of Tübingen	76:**78**:83	90:**91**:101	87:**89**:92	92:**94**:99
University of Edinburgh	30:**30**:31	29:**30**:31	28:**29**:30	26:**27**:30	University of Groningen	74:**80**:91	69:**73**:84	74:**79**:89	78:**82**:87
University of Melbourne	31:**31**:32	32:**32**:34	31:**32**:33	31:**32**:32	HUB	70:**80**:93	67:**73**:95	63:**67**:76	60:**62**:69
LMU Munich	32:**32**:33	32:**32**:33	32:**32**:33	34:**34**:36	OSUMC	78:**80**:82	68:**70**:72	71:**71**:74	69:**70**:73
UoCSD	32:**33**:34	29:**31**:32	29:**30**:31	28:**31**:31	University of Freiburg	80:**83**:88	87:**85**:92	76:**76**:80	81:**82**:84
UoBC	35:**34**:36	34:**34**:37	36:**37**:39	33:**34**:37	University of Copenhagen	82:**84**:96	91:**101**:105	110:**116**:130	103:**109**:124
King's College London	34:**35**:39	35:**36**:39	37:**38**:40	35:**36**:39	Emory University	78:**85**:102	72:**80**:96	75:**84**:97	84:**98**:111
Karolinska Institute	35:**36**:40	37:**41**:44	39:**40**:44	36:**38**:40	University of Minnesota	84:**85**:93	78:**79**:85	69:**71**:76	55:**56**:60
The University of Tokyo	29:**36**:48	30:**36**:49	35:**43**:55	38:**45**:56	École Polytechnique	85:**87**:102	86:**93**:109	99:**108**:128	107:**115**:124
GIoT	37:**38**:38	37:**38**:39	32:**34**:36	31:**33**:34	UoSaToC	79:**87**:102	71:**80**:96	79:**92**:102	115:**132**:142
University of Hong Kong	36:**39**:41	35:**35**:40	35:**36**:41	34:**40**:42	Sorbonne University	83:**87**:101	78:**80**:91	69:**73**:82	N/A
McGill University	38:**40**:40	41:**42**:42	43:**45**:46	42:**42**:44	UoMCP	88:**90**:97	89:**91**:97	81:**83**:88	66:**69**:72
TUoM	40:**41**:42	41:**43**:44	43:**45**:45	41:**41**:43	University of Bristol	84:**91**:104	77:**87**:101	70:**77**:90	69:**76**:79
Heidelberg University	38:**42**:47	40:**44**:49	43:**47**:50	43:**45**:48	University of Basel	86:**92**:105	86:**94**:107	98:**103**:109	90:**95**:106
EPFdL	41:**43**:45	37:**38**:39	34:**35**:37	36:**38**:41	University of Glasgow	85:**92**:103	87:**98**:109	82:**92**:103	76:**80**:87
University of Texas at Austin	39:**44**:45	35:**38**:41	36:**39**:41	46:**49**:50	PUWL	81:**94**:127	74:**88**:117	61:**64**:73	53:**60**:67
KU Leuven	43:**45**:46	45:**45**:48	48:**48**:50	46:**47**:49	Zhejiang University	74:**94**:144	74:**107**:151	67:**101**:143	127:**177**:234
Paris Sciences et Lettres	41:**46**:48	42:**45**:50	39:**42**:45	70:**72**:75	KAIST	79:**96**:129	83:**110**:137	83:**102**:117	84:**95**:111
NTUS	45:**47**:51	45:**49**:49	48:**51**:53	49:**52**:55	NTU	87:**97**:120	99:**120**:146	138:**171**:206	166:**198**:246
UoIaUC	47:**48**:53	44:**48**:49	48:**50**:52	36:**37**:39	UoCI	89:**98**:110	83:**96**:109	83:**96**:102	90:**99**:109
UoWM	50:**49**:54	47:**51**:51	41:**44**:46	41:**43**:45	University of Helsinki	93:**98**:104	91:**96**:102	93:**99**:100	89:**90**:94
WUiSL	46:**50**:54	47:**52**:54	51:**55**:57	47:**50**:53	SJTU	79:**100**:150	111:**157**:218	133:**188**:247	142:**188**:252

California Institute of Technology (CalTech), Massachusetts Institute of Technology (MIT), University of California – Berkeley (UoCB), University of California – Los Angeles (UoCLA), University of Michigan-Ann Arbor (UoMAA), National University of Singapore (NUoS), London School of Economics and Political Science (LSoEaPS), University of California – San Diego (UoCSD), University of British Columbia (UoBC), Georgia Institute of Technology (GIoT), Technical University of Munich (TUoM), École Polytechnique Fédérale de Lausanne (EPFdL), Nanyang Technological University – Singapore (NTUS), University of Illinois at Urbana-Champaign (UoIaUC), University of Wisconsin-Madison (UoWM), Washington University in St Louis (WUiSL), University of Manchester (UoM), University of Southern California (UoSC), Chinese University of Hong Kong (CUoHK), The Hong Kong University of Science and Technology (THKUoSaT), University of North Carolina at Chapel Hill (UoNCaCH), Australian National University (ANL), Seoul National University (SNU), The University of Queensland (TUoQ), Wageningen University & Research (WU&R), University of California – Davis (UoCD), University of California – Santa Barbara (UoCSB), Erasmus University Rotterdam (EUR), Charité – Universitätsmedizin Berlin (CUB), Delft University of Technology (DUoT), Humboldt University of Berlin (HUB), Ohio State University – Main campus (OSUMC), University of Science and Technology of China (UoSaToC), University of Maryland – College Park (UoMCP), Purdue University West Lafayette (PUWL), Korea Advanced Institute of Science and Technology (KAIST), National Taiwan University (NTU), University of California – Irvine (UoCI), Shanghai Jiao Tong University (SJTU)

[25] Lynch, K. Control by numbers: new managerialism and ranking in higher education. *Critical Studies in Education 56* (2015), 190–207.

[26] Marginson, S. University rankings and social science. *European Journal of Education 49* (2014), 45–59.

[27] Moed, H. A critical comparative analysis of five world university rankings. *Scientometrics 110* (2017), 967–990.

[28] Olcay, G., and Bulu, M. Is measuring the knowledge creation of universities possible?: A review of university rankings. *Technological Forecasting and Social Change 123* (2017), 153–160.

[29] Paruolo, P., Saisana, M., and Saltelli, A. Ratings and rankings: voodoo or science? *Journal of the Royal Statistical Society Series A (Statistics in Society) 176* (2013), 609–634.

[30] Piro, F., and Sivertsen, G. How can differences in international university rankings be explained? *Scientometrics 109* (2016), 2263–2278.

[31] Rahnamayan, S., Mahdavi, S., Deb, K., and Bidgoli, A. Ranking multi-metric scientific achievements using a concept of pareto optimality. *Mathematics 8* (2020), 956.

[32] Saisana, M., d'Hombres, B., and Saltelli, A. Rickety numbers: Volatility of university rankings and policy implications. *Research Policy 40* (2011), 165–177.

[33] Soh, K. Misleading university rankings: cause and cure for discrepancies between nominal and attained weights. *Journal of Higher Education Policy and Management 35* (2013), 206–214.

[34] Soh, K. What the overall doesn't tell about world university rankings: examples from arwu, qswur, and thewur in 2013. *Journal of Higher Education Policy and Management 37* (2015), 295–307.

[35] Waltman, L., Calero-Medina, C., Kosten, J., Noyons, E., Tijssen, R., van Eck, N., van Leeuwen, T., van Raan, A., Visser, M., and Wouters, P. The leiden ranking 2011/2012: Data collection, indicators, and interpretation. *Journal of the American Society for Information Science and Technology 63* (2012), 2419–2432.

[36] Williams, H. *Logic and Integer Programming.* Springer, Verlag, 2009.

[37] Wu, H.-Y., Chen, J.-K., Chen, I.-S., and Zhuo, H.-H. Ranking universities based on performance evaluation by a hybrid mcdm model. *Measurement 45* (2012), 856–880.