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A Simulation of Optimal Model on Fractional Aircraft Ownership (FAO) Management

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Abstract

Compared to owning a private jet, Fractional Aircraft Ownership (FAO) concept is a cheaper alternative for very mobile business persons who want to travel in comfort. The aircraft is owned by a number of customers (referred to as "owners") and the flight hours of its operation are shared based on each owner's portion. In this research, we do the simulation of an FAO company with very large demands with 27 cities of destination, which are commonly visited by business people in Indonesia. We derive flight demands stochastically from the owners and create optimal flying schedules based on the demands. Using the calculation of fixed and variable costs, we can determine the optimal flight pairings that minimized the operational cost. Eventually, we can determine the number of aircraft needed to be owned by FAO so the business will profit.

Keywords: *Optimization, Flight Scheduling, Stochastic Simulation, Aviation Industry, Mathematical Modelling, Investment.*

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1 Introduction

Traveling by airplane is an effective and efficient form of transportation in reaching cities that are far from each other. Especially in an archipelagic country such as Indonesia consisting of 17,504 islands. Problems of aviation industry has been widely taken up as research problems in order to find optimal ways in operating the related business. In [4], the survey made among 249 airline businesses showed significant differences in terms of risk and estimated cost of capital. A significant interaction between investment analysis and the way projects were financed was found, where airlines did not seem to use the most advanced technology on the market very often in spite of more sophisticated technologies being used.

Due to observation showing that many large fractional jet airlines had not been profitable, the authors in [13] discussed various strategic planning issues, such as aircraft maintenance, staff turnover, demand growth, and differentiation. Their impact on resource utilization and profitability were analyzed. Using the column generation procedure, the pricing problem by finding the shortest path in each crew network was solved in 1,2,3-day planning horizons respectively.

Some numerical methods are commonly utilized for solving optimization problems. Having implemented the method of Simulated Annealing using data of Garuda Indonesia, a national airline company in Indonesia, paper [12] solved the aircrew-assignment problem and its computational aspects that served 42 domestic and international destinations. The results showed the minimum number of the aircrew needed and its optimal allocation for operating the flights that balanced the flying and duty hours for each crew.

Authors in [6] derived mathematical expressions for the cockpit crew labor regulations and solved the optimization problem of nonlinear integer programming for finding the minimum value of mean relative deviations of the total flight time from the ideal flight time. The data being used was crew classes in the cockpit of Garuda Indonesia and the method being used is the simulated annealing method.

In [8], a goal programming of selecting optimal pairings covering all provided flights was solved by heuristic method like Bat Algorithm (BA), which was mimicking the bat behaviour, so the operational cost such as crew cost could be minimized.

A modification of the optimization problem could be made so it would propose a more realistic solution. In [11], the aircrew assignment problem was solved with constraints derived from the implemented regulations, such as flying time, resting time, the total number of takeoffs, and the number of holidays and workdays. Data being used was of a one-month full flight schedule from a big airline in Indonesia. Using a simple fuzzy logic approach, the paper proposed to find a new flying time tweaked from the existing regulation as in [12], so it can have better results on the personnel cost and evenly distribute the assignments.

A recent paper [5] proposed an optimization model

that can be used as a decision support tool for employees of small and medium-sized airlines. It can be used when operational rescheduling the work of flight crews is required due to an emergency arising, instead of using an intuitive approach that could lead to inaccuracies. So unnecessary financial losses can be avoided. Despite having regular commercial flights for transportation, business persons that are required to be actively mobile might accept the Fractional Aircraft Ownership (FAO) concept, which is joint aircraft ownership among a number of people by dividing the use of flight hours based on their portion of shares. It means an individual or a company can have a private jet without paying the total price of the aircraft. The owners of shares can book the plane for any journey as long as the availability of their flight hours. Research in FAO management has similar yet different from the ones for regular flight companies.

In [10], the demands of FAO owners were generated for flying among 8 (eight) airports. Optimal flight assignments using the Column Generation method were formulated to minimize the required number of aircraft so the profit was maximized and the daily operating cost was minimized.

Using data from a fractional management airline company operating in European and Asian countries, paper [7] proposed an optimization model to support the decision-making process involving the positioning of aircraft, which was not available at the requested airports of customer departure. The objective was to make this positioning cost be as low as possible. The model also provided more freedom in decision-making by making predictions of flight delays and maintenance events within a certain tolerance.

In this research, we develop a model of FAO management and its implementation using data in Indonesia with very large demands using 27 cities of destination. A big question being asked is whether this model can become a good investment or not. A simulation of this model is conducted to find optimal conditions to make the investment profitable.

2 Fractional Aircraft Ownership (FAO)

At the beginning of a period, all shareholders or owners must sign a contract with the FAO Company, which is valid for a particular period, for example, five years. The company provides a total flight time of h hours for one year, for example $h = 800$. This flight hour can be purchased in multiples of 50 hours so that the smallest share sold is $50/h$. If an owner needs a high frequency of flying, he/she should buy larger shares to get more flight time. The owner needs to pay a fixed monthly maintenance fee and the non-fixed operating fee. In this research, in order to maximize the occupancy of flying time, shareholders must provide FAO management with one month's plan request ahead. However, they can have an alteration of the plan with a notice in advance. We assume it is not possible for the company to serve more than one owner in one aircraft.

Sometimes, FAO could be overwhelmed by the owners requests and the existing aircraft has been full-occupied at the same time. FAO management has an obligation to serve all requests if the owners still have their right. The management should outsource the request to another private jet company and the cost, which is more expensive, is paid by the management. Therefore, there is a question on how many aircraft that should be owned by the management so the risk of deficit due to outsource expenses will be at lowest.

In this paper, firstly we develop a method for generating random requests from the owners. Having had the list of requested routes on monthly bases, the allocation of optimal flight pairs is constructed so the flight operational cost will be optimal using the plane owned by FAO management. If not all requests can be served by this plane, FAO needs to outsource by renting other planes from other private jet rental companies. So there is also a question of whether the FAO needs to have more than one plane because the outsourcing cost will make the operational expenses higher. The next step is to calculate the income and expense of the FAO so the rate of the investment's return can be determined. The number of owners will be simulated so we can determine this optimal number with the highest rate of return.

The scheduling in the FAO system is different from the scheduling of commercial airlines. A flight request in FAO At the beginning of a period, all shareholders or owners must sign a contract with the FAO Company, which is valid for a particular period, for example, five years. The company provides a total flight time of hours for one year, for example . This flight hour can be purchased in multiples of 50 hours so that the smallest share sold is . If an owner needs a high frequency of flying, he/she should buy larger shares to get more flight time. The owner needs to pay a fixed monthly maintenance fee and the non-fixed operating fee. In this research, in order to maximize the occupancy of flying time, shareholders must provide FAO management with one month's plan request ahead. However, they can have an alteration of the plan with a notice in advance. We assume it is not possible for the company to serve more than one owner in one aircraft. In this paper, firstly we develop a method for generating random requests from the owners. Having had the list of requested routes on monthly bases, the allocation of optimal flight pairs is constructed so the flight operational cost will be optimal using the plane owned by FAO management. If not all requests can be served by this plane, FAO needs to outsource by renting other planes from other private jet rental companies. So there is also a question of whether the FAO needs to have more than one plane because the outsourcing cost will make the operational expenses higher. The next step is to calculate the income and expense of the FAO so the rate of the investment's return can be determined. The number of owners will be simulated so we can determine this optimal number with the high-

est rate of return. The scheduling in the FAO system is different from the scheduling of commercial airlines. A flight request in FAO.

3 Generating Stochastic Demands

To make it as realistic as possible, demands of flight per day cannot be set as constant number. We need to generate stochastic flying schedule per owner containing the time and the cities being visited.

3.1 Amount of flying time per owner per month

Assume that the available ownership is fully sold. Let total flight time $h = 800$,n is the number of shareholders, $2 \leq n \leq 16$, and α_i is is the share portion of owner$-i$ that is multiples of $\frac{1}{16}$, $\frac{1}{16} \leq \alpha_i < 1, i = 1, 2, \ldots, n$.

$$\sum_{i=1}^{n} \alpha_i = 1.$$

The amount of flying time (hours) for owner$-i$ is

$$x_i = \alpha_1 h, \sum_{i=1}^{n} x_i = h.$$

The value above is for one year, so we need the flying time per month. For each owner, this value would be divided by 12 if there were no reference information on monthly bases. Heuristically, there are months when people fly more frequently than the rest of the months in a year. Therefore, we assume there is a proportion value for each month that quantifies the favorable time in a year. It is assumed that the proportions $\rho_k^m, k = 1, 2, \ldots, 12$ have values defined heuristically. For month$-k$, the amount of flying time (hours per month) of owner$-i$ is

$$x_{i,k} = x_i \rho_k^m, \sum_{i=1}^{12} x_{ik} = x_i, i = 1, 2, \ldots, n. \quad (1)$$

In generating the detail of owner's request in hour per day, the Poisson distribution is used with the parameter

$$\lambda = \frac{p_k}{x_{ik}} \times 24$$

where p_k is the number of days in the k-th month.

3.2 Preferences on the more popular routes

The company has an airport as the base, which means the first departing airport and the last destination airport of the day is the base. Denote m be the number of airports. In this research, $m = 27$ is the number of chosen airports in Indonesia that are regularly visited by business people, and the base airport is Soekarno-Hatta Airport (CGK). From these airports, we generate a number of couples of airports that determine the departure and arrival airports, so there are $m(m-1)$ routes from any two airports.

There are routes that are more popular than others, so a probability portion is given to each route that is required in generating the stochastic owner's request. To provide the portions, we use historical data on the number of passengers who arrive at and depart from each airport. We assume that the larger number of historical passengers the more popular the airport, and consequently the higher the probability portion.

Let α_j and d_j be be numbers of passengers who respectively arrive at and depart from airport$-j$ for a year. Let ρ_{jk}^{p1} be the proportion describing the popularity of the route from airport$-j$ to airport$-k$. Assume this proportion is applicable for any period of time, for instance, year and month. Its value is defined by the multiplication of the ratio of departures from airport$-j$ as follows

$$\delta_j^d = \frac{d_j}{\sum_{i=1}^{27} d_j},$$

and the ratio of arrivals at another airport$-k$ as follows,

$$\delta_k^a = \frac{\alpha_k}{\sigma_{i=1}^{27} \alpha_k}, k \neq j,$$

then the multiplication is divided by the total sum of all these multiplications. The formula is following

$$\rho_{jk}^{p1} = \frac{\delta_j^d \delta_j^a}{\sum_{i=1}^{27} \sum_{k=1,k\neq j}^{27} \delta_j^d \delta_j^a}, k \neq j. \quad (2)$$

For simplicity, these proportions of routes are named in order indices by

$$\tilde{\rho}_j^{p1}, j = 1, 2, \ldots, m(m-1). \quad (3)$$

After flying from the first airport to the second airport, most of the owners will go back to the first airport. An owner can have a request to fly to the third airport with small probability ω_1. In this research we choose $\omega_1 = 30\%$. We determine the second preference for airports to be the third airport as the destination with the following formula.

$$\rho_l^{p2} = \frac{\delta_l^a}{\sum_{j=1}^{27} \delta_j^a}, l = 2, 3, \ldots, m. \quad (4)$$

Now we arrange the last flight of the day by defining ω_2 as the probability that the owner flies back to the first airport and $1 - \omega_2$ probability that the owner flies back to the second airport. Here $\omega_2 > 1 - \omega_2$, where $\omega_2 = 75\%$ in this research.

4 Possible Pairings

We develop groups of possible flight pairing, which are flight schedules containing routes whose the first departure and the last destination are in the airport base, CGK. This is a collection of the routes that will be served by one aircraft departing from and going back to the base in order to serve some requests on a particular day. If there are many requests so there will be many possible pairings formed. The types of pairings can be seen in Table 1 and Figures 1 to3 . Note that there are constraints to be fulfilled for one day, those are maximum of 8 hours flying time and of 14 hours

duty time for airline staffs. So pairing A_6 contains the maximum number of routes. Some specific routes need more than 8 hours flying time, so they cannot be combined with ordinary routes. Those pairings are defined in K_1 and K_2, whose forms are similar to A_1 and A_2.

Figure 1: Pairings A_2 (left) and A_3 (right).

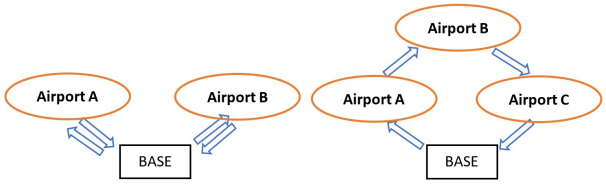

Figure 2: Two types of pairings A_4.

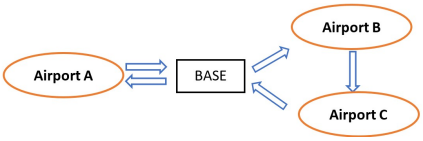

Figure 3: Pairing A_5.

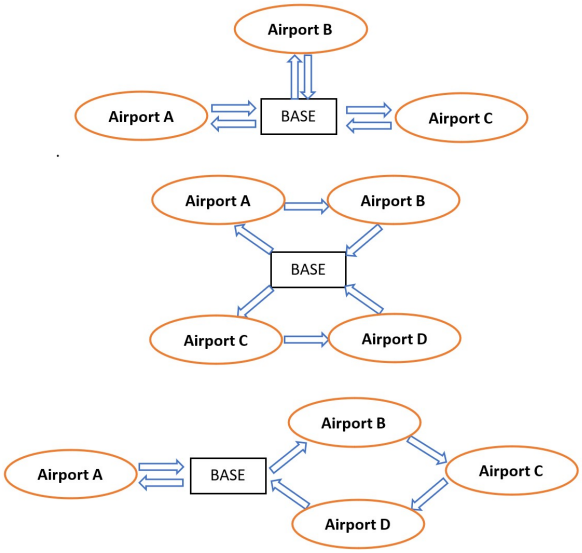

Figure 4: Three types of Pairing A_5.

Matrix $A_s, s = 2, 3, 4, 5, 6$ is of size $nr \times na_s$, where nr and na_s are the numbers of routes in a day and available pairings of type$-s$, respectively. Matrix K_s has dimension $nr \times nk_s, s = 1, 2$, where nk_s is available pairings of type K_s. On each column, the matrix has value 1 for s entries and value 0 for the remaining entries. Let a^s_{ij} be the component of matrix A_i, where $a^s_{i_w,j} = 1, w = 1, 2, \ldots, s$ if route i_w is chosen for pairing$-j$ containing CGK, as the base, the owners' requested airports or possible airports for Deadhead flights. A Deadhead flight is a flight without an owner as the passenger. All these pairings form the possibility matrix A as below

$$A = [A_2 A_3 A_4 A_5 K_1 K_2]\,. \tag{5}$$

Table 1: Types of possible pairings

Pairings	Route(s)
A_2	Base-airport A - Base
A_3	Base – Airport A – Airport B – Base
A_4	Base – Airport A – Base – Airport B – Base (Combination of 2 of pairing), Base – Airport A – Airport B – Airport C – Base
A_5	Base – Airport A – Base – Airport B – Airport C – Base (Combination of pairings A_2 and A_3 or otherwise)
A_6	Combination of 3 of pairings A_2, or 2 of pairings A_3, or pairings A_2 and A_4
K_1	Similar to A_2 but the flying time is more than 8 hours
K_2	Similar to A_3 but the flying time is more than 8 hours

Matrix A has dimension $nr \times np$, where

$$np = na_2 + na_3 + na_4 + na_5 + na_6 + nk_1 + nk_2.$$

We will determine the optimal pairing by using the following optimization model. Let x_j be a binary decision variable, $x_j \in \{0, 1\}$. In this case, $x_j = 1$ means the j-th pairing is chosen and $x_j = 0$ means the j-th pairing is not chosen. We define parameters for this model based on the operational cost of $j-$th pairing, denotes by c_j . Other parameters is defined based on the entries of matrix A. Let $\bar{a_{ij}}$ and $\bar{a_j}$ be respectively the component and column vector of $A, i = 1, 2, \ldots, nr, j = 1, 2, \ldots .np$. Optimal pairings are found by solving the following problem.

$$Min \sum_{j=1}^{np} c_j x_j. \tag{6}$$

And this problem must satisfy this following constraint

$$\sum_{i=1}^{np} \bar{a}_{i^r j} x_{i^r} = 1, \sum_{i=1}^{np} \bar{a}_{i^d j} x_{i^d} \leq 1 \tag{7}$$

for all i^r is the indices of requested route by owners, and all i^d is the indices of Deadhead flights. The optimization problem of finding the optimal pairings will be solved by Balas' algorithm [9] for the zero-one integer linear programming problem.

4.1 An illustration

Table 2 shows an illustrative example containing requests from three owners for a day. Owner 1 request one way flight at 00:00 in the morning from Pekanbaru (PKU) to Banten (CGK) with flying time 2 hours and 1 minute. Owner 2 requests round trip Denpasar (DPS) – Palembang (PLM) with each flying time 2 hours 35 minutes at different time but in the same day. Owner 3 also requests a round trip Banten (CGK) – Banjarbaru (BDJ) with each flying time 1 hour 59 minutes. Note that FAO aircraft firstly departs from and finally arrive to the base airport, which is CGK. In Table 3,

all possible routes are developed. The first column is the route number and the owner number. The Deadhead flights is named owner 0. The departure time of route-1 is 00:00, so the aircraft begins to serve on the previous day (D-1) at 21:24. The arrival time of route-14 is 01:35 in the next morning, so the aircraft will arrive at CGK on the next day (D+1).

Table 2: Illustrative requests from 3 owners

Owner	Dep Time	Fly Time	Arr Time	Dep Airpt	Arr Airpt
1	00:00	02:01	02:01	PKU	CGK
2	03:00	02:35	05:35	DPS	PLM
3	16:00	01:59	17:59	CGK	BDJ
3	20:00	01:59	21:59	BDJ	CGK
2	23:00	02:35	01:35	PLM	DPS

Table 3: Illustrative possible routes

Route/ Owner	Dep Time	Fly Time	Arr Time	Dep Airpt	Arr Airpt
1(0)	21:24 (D-1)	02:01	23:25 (D-1)	CGK	PKU
2(1)	00:00	02:01	02:01	PKU	CGK
3(0)	00:26	01:59	02:25	CGK	DPS
4(2)	03:00	02:35	05:35	DPS	PLM
5(0)	06:10	01:08	07:18	PLM	CGK
6(0)	06:10	02:10	08:20	PLM	BDJ
7(3)	16:00	01:59	17:59	CGK	BDJ
8(0)	17:26	01:59	19:25	CGK	BDJ
9(0)	18:34	01:59	20:33	BDJ	CGK
10(0)	18:34	02:10	20:44	BDJ	PLM
11(3)	20:00	01:59	21:59	BDJ	CGK
12(0)	21:17	01:08	22:25	CGK	PLM
13(2)	23:00	02:35	01:35 (D+1)	PLM	DPS
14(0)	02:10 (D+1)	01:56	04:06 (D+1)	DPS	CGK

Requests from owners 1, 3 and 4 depart from non-base airport, so all possible Deadhead flights are developed based on the time needed before or after flights of the owners' requests. For example in serving owner 1, an aircraft is needed to be in PKU at 00:00, so FAO sends this aircraft from CGK to PKU with the arrival time 35 minutes before the next take-off at 00:00, in order to do reporting.

Furthermore in Table 3, there are routes automatically developed which are possible but it might be inefficient. For examples, route 6 is developed directly after route 4, so PLM is the destination airport, and it is due to the request of owner 3 of route 11, so BDJ is the destination airport. Route 10 is also developed due to route 7, so the departure airport is BDJ, and route 13, which is the owner-2 request. Intuitively, these later routes could be omitted from the table if the list in table is not too long. If the list is long and complex with overlapping times, these routes could give more possible optimal pairings. For Table 3, eventually routes 6 and 10 are not chosen for pairings because there are other routes that make more efficient pairings.

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{8}$$

$$A_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \quad A_4 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{9}$$

We develop possible pairings based on types in Table 3. There are four pairings of type A_2, which are for CGK – PKU – CGK (routes 1 – 2) and for CGK – BDJ – CGK (routes 7 – 9, 7 – 11, and 8 – 11). There are 2 pairings of type A_3, which are for CGK – DPS – PLM – CGK (routes 3 – 4 – 5) and for CGK – PLM – DPS – CGK (routes 12 – 13 – 14). Pairings for CGK – PKU – CGK – BDJ – CGK with type A_4 are the combinations of type A_2, which are routes 1 – 2 – 7 – 9, routes 1 – 2 – 7 – 11, and 1 – 2 – 8 – 11. The matrices A_2, A_3 and A_4 are shown in equations (8) and (9).

In constructing matrix A in (5), matrices A_5, A_6, K_1 and K_2 are zeros matrices. By solving the optimization problem (6), we will find some optimal pairings, but we need to find the operational cost parameter first. In the next section, the step to find operational cost will be elaborated.

5 Valuation of FAO Investment

5.1 Income and expenses

For owner$-i$ where $i = 1, 2, \ldots .n$, FAO receives income that consist of the ownership fee OF_i, monthly management fee MF_i and occupied hourly fee HF_i. FAO's expenses include the operational cost, the aircraft maintenance cost, the insurance cost and others. The cost

$OF_i(k)$ is the paid annually and calculated from the aircraft's share-price of owner$-i$ and depended on the number of aircraft being utilized q,

$$OF_i(q) = \frac{1}{5}\alpha_i q P_1,$$

where P_1 is the price of one aircraft. Each owner needs to pay it in 5 years, because the ownership contract is 5 years. The cost MF_i is paid monthly for maintenance cost, insurance, salary for pilots and aircraft staffs. The amount of this cost is proportional to the share owned. Let P_2 is unit cost USD for management per share owned, then

$$MF_i = 16\alpha_i P_2.$$

The cost HF_i is paid annually for paying the hours being occupied for fuel and operational cost. Let P_3 be the cost rate USD per hour.

$$HF_i = h\alpha_i P_3.$$

The yearly income of FAO is

$$\begin{aligned} I(q) &= \sum_{i=1}^{n} OF_i(q) + 12MF_i + HF_i \\ &= \sum_{i=1}^{n} \alpha_1 \left(\frac{q}{5}P_1 + 12(16)P_2 + hP_3\right) \end{aligned} \tag{10}$$

FAO spends 2 types of expenses; fixed cost T_1 and variable cost T_2 depending on the flying time FT. The fixed cost T_1 only depends on the number of aircraft owned by FAO, where are based on [3]. Total fixed cost is defined by

$$FC(q) = qT_1. \tag{11}$$

The operational cost OP per day consist of the Parking Cost Pr and Landing Cost Ld at all discussed airports. For some routes, aircraft sometimes cannot fly directly to their destination, and must make a transit if the distance traveled exceeds the cruising range. There are additional cost of transit TC, consisting of parking cost and landing cost. Therefore, the operational cost of a pairing p on a certain day d can be determined using the following formula

$$OP_{pd} = T_2\left(FT_{pd}\right) + Pr_{pd} + Ld_{pd} + TC_{pd}. \tag{12}$$

The other component of the variable cost is outsourcing cost when the owned aircraft will be over-occupied by the owners requests. Let OTC_d be the outsourcing cost which is the rental price of an aircraft from other companies at day d. The unit cost is assumed to be the same for all outsourcing companies, which is $Rt = 3,350$ USD per hour of the flying time. The outsourcing cost is defined by

$$OTC_d = FTO_d \times Rt, \tag{13}$$

where FTO_d in hour(s) is the remaining flying time that cannot be covered by the usage of q existing aircraft. Here we choose the pairing that served by outsourcing aircraft such that its flying time is the lowest, because the cost for rent an aircraft from an outsourcing company is more expensive than the operational cost for flying the company's owned aircraft.

Finally we can define total of cost in a year by this following equation

$$C(q) = FC(q) + \sum_{d=1}^{365}\left(\sum_{\tilde{p}=1}^{n\tilde{p}} OP_{\tilde{p}d} + OTC_d\right). \tag{14}$$

Note that $\sum_{\tilde{p}=1}^{n\tilde{p}} OP_{\tilde{p}d}$ is the total cost of the optimal pairing in day d.

At the end of year 5, we will evaluate the price of the existing aircraft in order to know the final value of the asset of FAO. Based on Airline Disclosure Guide, generally aircraft assets are depreciated over 15 to 25 years with residual values between 0 to 20 percent. Suppose we take the median, so it means the price is depreciated over 20 years with residual values of 10 percent. If the sale price of an aircraft is P_1 when it is bought at the beginning of year 1, then the estimated yearly depreciation cost is equal

$$D = \frac{P_1 - 10\%P_1}{20} = 0.045P_1. \tag{15}$$

If it is assumed that the depreciation goes as in a decreasing line, so at the end of year 5, the price of aircraft will become $P_1 - 5D$.

5.2 Rate of return

To value an investment whether it is profitable or not, one of the observable indicators is the rate of return r. Commonly if this rate is higher than the inflation rate, it is considered a good investment. In FAO investment, efficient pairings of routes to serve the daily request of owners will reduce the cost. The highest expense in this investment is buying the aircraft. Therefore, we simulate the model of FAO defined in the previous sections in order to answer the optimal number of the aircraft.

Let $R(q, r)$ be a function containing the number of aircraft q and the rate of return r. Let $C_j(q)$ be the total cost of the year. It is assumed that the annual income is paid at the beginning of the year, and the operational cost is recorded at the end of the year. The rate of return r is the desired solution or the root of the equation $R(q, r) = 0$.

$$\begin{aligned} R(q,r) = I(q) - qP_1 + \sum_{j=1}^{4}\frac{I(q) - C_j(q)}{(1+r)^j} \\ + \frac{q(P_1 - 5D) - C_5(q)}{(1+r)^5} \end{aligned} \tag{16}$$

Let $\tilde{R}(q)$ be kind of inverse function with respect to r. We can write the optimisation problem of investment is

$$Max_q r = \tilde{R}(q). \tag{17}$$

It is only possible to estimate the solution of problem (16) numerically, by using the root finding method. Solutions to the problem (17) are concluded from the results of the simulations.

6 Numerical Simulation

Implementation of the FAO model is using data in Indonesia, but the chosen currency is USD to make financial data easier to write. The type of aircraft being used is Phenom 300 with 6 to 8 passengers, where the maximum speed is 859 km/hour, and the weight is 6350 kg [2, 1]. It has been used at an average speed of 80% of its maximum speed. We conduct simulations for the total flight time of hours to be 800 and 640 hours.

Using the number of airports $m = 27$, the generated number of routes is 702 routes, where the airplane flies less than or equal to 8 hours per day. Due to this limitation on a single flying time, there are only 2 cities that can be served for one request. Some routes have a flying time more than the limit so there should be a transit airport between the departure and destination airports.

Table 4: Data for Income (10)

Item	Cost (USD)
P_1	8,760,000
P_2	7,832
P_3	1,566

In this simulation, we assumed that there are 5 owners who had owner share as follows:

$$S_1 = \frac{2}{16}, S_2 = \frac{2}{16}, S_3 = \frac{3}{16}, S_4 = \frac{4}{16}, S_5 = \frac{5}{16}$$

The parameter values in equation (10) are shown in the Table 4. The fixed and variable costs are written in Tables 5 and 6.

Table 5: Data for Fixed Cost (11)

Item	Cost (USD)
Crew Salaries	208,000
Hangar	29,700
Insurance	32,888
Recurrent Training	26,200
Modernization	20,000
Navigation Chart Service	3,742
Refurbishing	18,900
Computer Maintenance Program	3,250
Weather Service	700
Total (T_1)	343,380

Table 6: Data for variable Cost (12)

Item	Cost (USD)
Fuel	787
Maintenance Labor	68
Engine Restoration	92
Crew Expenses	70
Supplies	33
Total (T_2)	1030

Based on the simulations, the average expected total cost $C(q)$ per year in USD is shown in Figure 5, where the management has q aircraft. The cost for the number of aircraft $q = 1$ tends to be the highest among the others. Because there are rental expenses of some outsourcing aircraft that must be provided to serve the owner's requests. For other values of q , we can see that the total cost is a little bit decreasing for $q = 2$ and $q = 3$. Furthermore, the cost tends to increase for the number of aircraft $q = 4$ and $q = 5$. This is due to the increase of fixed costs that include the total price of all aircraft bought by FAO management. Based on the simulation result, the minimum cost is achieved when $q = 3$.

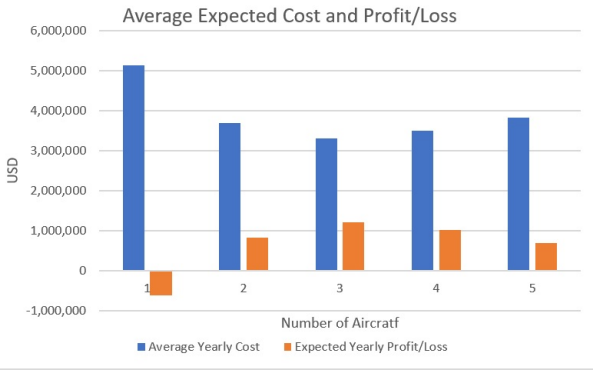

Figure 5: Average Cost and Profit per year

The average expected profit per year with q aircraft in USD based on the simulations is shown in Figure 5. The calculation is summation of the profit for 5 years, and it is not considering the time reference when the profit being produced. The maximum profit is expected when $q = 3$.

Table 7: Rate of return $h = 800$ hours

Nb of aircraft	ROR per year (%)
1	-31.754
2	0.308
3	0.125
4	-2.072
5	-3.665

Now we calculate the rate of return (ROR) per year using equation (16), where the result is shown in Table 7. If FAO has 1, 4, or 5 planes, then FAO management will suffer losses when fulfilling the owner's request. On the other hand, if FAO has 2 or 3 planes, then FAO will make a profit, with the biggest ROR for having 2 aircraft. So the optimal number of aircraft that FAO must have is 2 aircraft. However, the ROR is very small so it will discourage investors to make FAO as their business.

Now using the same income, we consider having lesser total flight time being committed, which is 640 hours per year. As shown in Table 8, the rate of return per year tends to increase when the total flight hours are reduced. This is because when the total flight hours are reduced, the number of requests from the owners will also decrease. As a result, daily variable costs will

Table 8: Rate of return $h = 640$ hours

Nb of aircraft	ROR per year (%)
1	2.213
2	6.607
3	2.194
4	-0.937
5	-2.805

decrease and this causes expected total cost also decreases. Reduction of the total cost can lead to increases in the profit and the rate of return. Table 8 shows the total flight time per year is 640 hours, and FAO owning 2 planes will expect to have a rate of return of 6.607% per year.

7 Conclusion

FAO is a concept of joint aircraft ownership among a number of business people. This research wants to estimate the profit FAO management if the joint aircraft charter scheme is implemented in Indonesian data with very large number of airports being observed. The simulation is run using the python programming language and using Google Collab. We want to use the GPU accelerator so that the computation time is faster.

In this research, a stochastic scheme has been successfully built to generate requests from FAO owners. To optimize cost and time, an optimization model to determine optimal pairing has also been successfully built. Based on the assumption and the calculations in the simulation, the number of aircraft that provided the optimum profit and rate of return is 2 aircraft. Having total flight hours of 800 hours per year, the expected rate of return is 0.308 %. For total flight hours of 640 hours, FAO will be able to get an expected rate of return of 6.607%.

In the future, the research can be continued with the different provisions of shares of ownership and find the optimal form of the type of ownership that makes the most profit among others. For the application in the real world, the FAO management could think of the appropriate total flight hours so the obtained profit is acceptable.

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