Table 5: Comparison between BIRECT-Vl, BIRECT-V, BIRECT-l, BIRECT, DIRECT-l, and DIRECT algorithms.

Problem	BIRECT-Vl		BIRECT-V		BIRECT-l		BIRECT		DIRECT-l		DIRECT	
No.	$f(\bar{x})$	f.eval.	$f(\bar{x})$	f.eval.	$f(\bar{x})$	f.eval.	$f(\bar{x})$	f.eval.	$f(\bar{x})$	f.eval.	$f(\bar{x})$	f.eval.
1	1.52×10^{-5}	218	1.52×10^{-5}	260	2.54×10^{-5}	176	2.54×10^{-5}	202	7.53×10^{-5}	**135**	7.53×10^{-5}	255
2	1.52×10^{-5}	524	1.52×10^{-5}	2728	2.54×10^{-5}	**454**	2.54×10^{-5}	1268	7.53×10^{-5}	1777	7.53×10^{-5}	8845
3	1.52×10^{-5}	1280	1.52×10^{-5}	137040	2.54×10^{-5}	**874**	2.54×10^{-5}	47792	3.57445	> 500000	7.53×10^{-5}	80927
4	8.77×10^{-5}	640	8.77×10^{-5}	1034	9.17×10^{-5}	436	9.17×10^{-5}	436	9.29×10^{-5}	**247**	9.29×10^{-5}	655
5	1.83×10^{-6}	254	3.17×10^{-5}	524	4.02×10^{-5}	468	4.02×10^{-5}	476	3.09×10^{-6}	**205**	3.09×10^{-5}	327
6	1.53×10^{-6}	252	1.53×10^{-6}	284	3.35×10^{-5}	472	3.35×10^{-5}	478	2.58×10^{-6}	**233**	2.58×10^{-5}	345
7	2.88×10^{-6}	284	2.88×10^{-6}	**282**	3.68×10^{-5}	474	3.67×10^{-5}	480	8.21×10^{-5}	573	8.21×10^{-5}	693
8	2.99×10^{-6}	300	2.99×10^{-6}	334	6.10×10^{-5}	**188**	6.10×10^{-5}	194	6.58×10^{-5}	215	6.58×10^{-5}	295
9	0.39791	656	0.39791	492	0.39790	242	0.39790	242	0.39789	**159**	0.39789	195
10	9.82×10^{-5}	2320	9.82×10^{-5}	1910	9.82×10^{-5}	**794**	9.82×10^{-5}	**794**	3.83×10^{-5}	3379	6.08×10^{-5}	6585
11	4.01×10^{-5}	810	4.01×10^{-5}	784	4.84×10^{-5}	722	4.84×10^{-5}	722	5.32×10^{-5}	**485**	6.25×10^{-5}	513
12	7.57×10^{-5}	10872	7.57×10^{-5}	8446	7.15×10^{-5}	**4060**	7.15×10^{-5}	**4060**	6.45×10^{-5}	54843	6.45×10^{-5}	19661
13	7.02×10^{-5}	**35492**	7.60×10^{-5}	50922	9.52×10^{-5}	162862	9.52×10^{-5}	164826	0.66667	> 500000	5.79×10^{-5}	372619
14	−0.99999	**180**	−0.99999	1082	−0.99999	480	−0.99999	16420	−0.99999	6851	−0.99999	32845
15	3.00000	**28**	3.00000	**28**	3.00019	274	3.00019	274	3.00009	115	3.00009	191
16	4.61×10^{-7}	8456	4.61×10^{-7}	9162	7.76×10^{-7}	**5106**	7.76×10^{-7}	**5106**	4.84×10^{-6}	8379	4.84×10^{-6}	9215
17	−3.86245	200	−3.86245	208	−3.86242	352	3.86242	352	−3.86245	**111**	−3.86245	199
18	−3.32214	542	−3.32214	542	−3.32206	764	3.32206	764	−3.32207	**295**	−3.32207	571
19	−1.03154	202	−1.03154	334	−1.03154	190	1.03154	334	−1.03162	**137**	−1.03162	321
20	9.03×10^{-6}	136	9.03×10^{-6}	154	9.09×10^{-5}	152	9.09×10^{-5}	152	2.10×10^{-5}	**77**	2.10×10^{-5}	105
21	1.83×10^{-5}	454	1.83×10^{-5}	558	1.83×10^{-5}	660	1.83×10^{-5}	1024	3.65×10^{-5}	**359**	3.65×10^{-5}	705
22	3.54×10^{-5}	**1182**	3.54×10^{-5}	7440	3.55×10^{-5}	1698	3.55×10^{-5}	7904	9.62×10^{-5}	5297	6.23×10^{-5}	5589
23	2.71×10^{-5}	148	2.71×10^{-5}	208	2.71×10^{-5}	90	2.71×10^{-5}	94	3.81×10^{-5}	**71**	3.81×10^{-5}	107
24	−1.80130	184	−1.80130	314	−1.80118	126	−1.80118	126	−1.80127	**45**	−1.80127	69
25	−4.68744	8430	−4.68744	**7472**	−4.68736	101942	−4.68736	73866	−4.68721	26341	−4.68721	13537
26	−8.60559	> 500000	−7.55576	> 500000	−7.32661	> 500000	−7.32661	> 500000	−7.84588	> 500000	−7.87910	> 500000
27	0.00132	> 500000	0.00189	> 500000	0.00203	> 500000	0.00203	> 500000	0.04054	> 500000	0.04355	> 500000
28	4.59×10^{-5}	2786	4.59×10^{-5}	**1678**	4.86×10^{-5}	1832	4.86×10^{-5}	2114	6.52×10^{-5}	32331	9.02×10^{-5}	14209
29	9.00×10^{-5}	**2872**	9.00×10^{-5}	3072	9.71×10^{-5}	92884	9.71×10^{-5}	99514	0.02488	> 500000	0.02142	> 500000
30	0.00000	**204**	9.97×10^{-5}	40788	9.00×10^{-5}	1718	9.00×10^{-5}	10856	0.03524	> 500000	0.00215	> 500000
31	4.81×10^{-5}	774	4.81×10^{-5}	958	4.81×10^{-5}	**154**	4.81×10^{-5}	180	2.30×10^{-5}	1727	2.30×10^{-5}	987
32	1.29×10^{-5}	9126	1.29×10^{-5}	11008	1.18×10^{-5}	**472**	1.18×10^{-5}	1394	4.97479	> 500000	4.97479	> 500000
33	1.98×10^{-5}	**124**	1.98×10^{-5}	1454	2.36×10^{-5}	1250	2.36×10^{-5}	40254	4.97479	> 500000	9.94967	> 500000
34	9.65×10^{-5}	698	9.65×10^{-5}	718	9.65×10^{-5}	**242**	9.65×10^{-5}	**242**	9.65×10^{-5}	285	9.65×10^{-5}	1621
35	2.41×10^{-5}	2444	2.41×10^{-5}	2972	2.41×10^{-5}	**1494**	2.41×10^{-5}	1700	5.75×10^{-5}	2703	8.80×10^{-5}	20025
36	3.05×10^{-5}	19134	3.05×10^{-5}	31430	5.42×10^{-5}	**4590**	5.42×10^{-5}	10910	8.29×10^{-5}	74071	8.29×10^{-5}	174529
37	1.37×10^{-7}	492	1.37×10^{-7}	564	5.64×10^{-5}	**210**	5.64×10^{-5}	236	2.88×10^{-5}	341	2.88×10^{-5}	255
38	3.42×10^{-7}	24272	3.42×10^{-7}	16704	6.41×10^{-5}	**1422**	6.41×10^{-5}	7210	7.21×10^{-5}	322039	7.21×10^{-5}	31999
39	1.77×10^{-8}	**1492**	1.77×10^{-8}	86306	1.30×10^{-6}	58058	1.30×10^{-6}	315960	1269.34444	> 500000	1187.63199	> 500000
40	−10.15234	6618	−10.15234	5866	−10.15234	1200	−10.15234	**147**	−10.152234	155		
41	−10.40201	2298	−10.40201	2604	−10.402269	1224	−10.40269	1180	−10.40196	**141**	−10.40196	145
42	−10.53544	2498	−10.53545	3324	−10.53618	1158	−10.53618	1140	−10.53539	**139**	−10.53539	145
43	−186.72944	**806**	−186.72945	1684	−186.72441	2114	−186.72441	1780	−186.72153	2043	−186.72153	2967
44	1.15×10^{-5}	112	1.15×10^{-5}	190	1.15×10^{-5}	108	1.15×10^{-5}	118	8.74×10^{-5}	**91**	8.74×10^{-5}	209
45	2.87×10^{-5}	392	2.87×10^{-5}	1400	2.87×10^{-5}	**288**	2.87×10^{-5}	712	7.49×10^{-5}	465	9.39×10^{-5}	4653
46	5.74×10^{-5}	1054	5.74×10^{-5}	27566	5.74×10^{-5}	**784**	5.74×10^{-5}	16974	9.63×10^{-5}	2057	6.32×10^{-5}	99123
47	8.74×10^{-5}	248	8.74×10^{-5}	280	7.94×10^{-6}	226	7.94×10^{-6}	244	3.53×10^{-5}	**77**	3.52×10^{-5}	107
48	3.97×10^{-5}	1354	3.97×10^{-5}	1776	3.97×10^{-5}	836	3.97×10^{-5}	1034	7.19×10^{-5}	**411**	7.19×10^{-5}	833
49	9.35×10^{-5}	3394	9.35×10^{-5}	9244	9.11×10^{-6}	3366	9.11×10^{-6}	7688	7.76×10^{-6}	**1809**	7.76×10^{-5}	8133
50	−49.99788	1312	−49.99788	1662	−49.99512	**1138**	−49.99512	1506	−49.99525	8731	−49.99525	5693
51	−209.98779	**3114**	−209.98779	11878	−209.98007	24716	−209.98007	30100	−209.92644	> 500000	−209.98085	90375
52	2.88×10^{-5}	**156**	2.88×10^{-5}	162	2.88×10^{-5}	338	2.88×10^{-5}	502	7.95×10^{-5}	209	7.95×10^{-5}	237
53	6.43×10^{-5}	**3810**	6.43×10^{-5}	4060	6.44×10^{-5}	27364	6.44×10^{-5}	20974	0.11921	> 500000	9.71×10^{-5}	316827
54	2.607286	> 500000	2.607286	> 500000	9.41133	> 500000	9.41133	> 500000	16.47703	> 500000	28.96394	> 500000
Average		**30844.296**		37072.0371		37283.85		44520.52		121484.19		98677.70
Median		808.00		1681.00		**789.00**		1190.00		1752.00		3810.00

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